State/Signal Linear Time-Invariant Systems Theory, Part I: Discrete Time Systems

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Abstract

This is the first paper in a series of several papers in which we develop a state/signal linear time-invariant systems theory. In this first part we shall present the general state/signal setting in discrete time. Our following papers will deal with conservative and passive state/signal systems in discrete time, the general state/signal setting in continuous time, and conservative and passive state/signal systems in continuous time, respectively. The state/signal theory that we develop differs from the standard input/state/output theory in the sense that do not distinguish between input signals and output signals, only between the "internal" states x and the "external" signals w. In the development of the general state/signal systems theory we take both the state space $\mathcal{X}$ and the signal space $\mathcal{W}$ to be Hilbert spaces. In later papers where we discuss conservative and passive systems we assume that the signal space $\mathcal{W}$ has an additional Kreĭn space structure. The definition of a state/signal system has been designed in such a way that to any state/signal system there exists at least one decomposition of the signal space $\mathcal{W}$ as the direct sum $\mathcal{W} = \mathcal{Y} \dotplus \mathcal{U}$ such that the evolution of the system can be described by the standard input/state/output systems of equations with input space $\mathcal{U}$ and output space $\mathcal{Y}$. (In a passive state/signal system may take $\mathcal{U}$ and $\mathcal{Y}$ to be the positive and negative parts, respectively, of a fundamental decomposition of the Kreĭn space $\mathcal{W}$.) Thus, to each state/signal system corresponds infinite many input/state/output systems constructed in the way described above. A state/signal system consists of a state/signal node and the set of trajectories generated by this node. A state/signal node is a triple $\Sigma = (V; \mathcal{X}, \mathcal{W})$, where V is a subspace with appropriate properties of the product space $\mathcal{X} \times \mathcal{X} \times \mathcal{W}$. In this first paper we extend standard input/state/output notions, such as existence and uniqueness of solutions, continuous dependence on initial data, observability, controllability, stabilizability, detectability, and minimality to the state/signal setting. Three classes of representations of state/signal systems are presented (one of which is the class of input/state/output representations), and the families of all the transfer functions of these representations are studied. We also discuss realizations of signal behaviors by state/signal systems, as well as dilations and compressions of these systems. (Duality will be discussed later in connection with passivity and conservativity.)

Keywords

State/signal node, driving variable, output nulling, input/state/output, linear fractional transformation, transfer function, behavior, external equivalence, realization, dilation, compression, outgoing invariant, strongly invariant, controllability, observability, minimality, stabilizability, detectability.