Regular Spectral Factorizations

Olof J. Staffans
Åbo Akademi University
Department of Mathematics
FIN-20500 Åbo, Finland
Olof.Staffans@abo.fi
http://www.abo.fi/˜staffans/

Ruth F. Curtain
Department of Mathematics
University of Groningen
P.O. Box 800
9700 AV Groningen, the Netherlands
R.F.Curtain@math.rug.nl

Problem 1 *Let X be an an invertible $n \times n$ matrix-valued function in the open right half plane, which belongs to H^∞ together with its inverse. Find conditions on the boundary function $\Pi(i\omega) = X(i\omega)^* X(i\omega)$ $(\omega \in \mathbf{R})$ which imply that the limit $E = \lim_{\lambda \to +\infty} X(\lambda)$ exists, and compute this limit. Here λ tends to infinity along the positive real axis.*

This open problem arose fairly recently out of some new results on infinite-dimensional regulator problems and the corresponding Riccati equations for the very general class of infinite-dimensional linear system known as (weakly) *regular well-posed linear systems*. The existence of solutions to Riccati equations is the key to solving many control problems and it is important to prove existence for as large a class of systems as possible. The approach used here is to reduce the regulator problem to the associated spectral factorization problem, appeal to known results for the existence of a spectral factor X, and then to use the beautiful properties of well-posed linear systems to construct a realization of this spectral factor from a given realization of the original system. If the spectral factor is regular, the appropriate algebraic Riccati equations can be derived, and the optimal solution can be constructed. The conclusions are perfect generalizations of the corresponding finite-dimensional conclusions, apart from the following gaps:

- Although, under the standard assumptions, the existence of a spectral factor is guaranteed, it need not be regular, i.e., the limit E need not exist. This regularity property is essential in the derivation of the Riccati equation.
- The resulting Riccati equation contains the operator $(E^*E)^{-1}$. Hence we need to compute E^*E, and E^*E must be invertible.

For example, if Π is given by

$$\Pi(i\omega) = R + NG(i\omega) + (NG(i\omega))^* + G(i\omega)^* QG(i\omega),$$

where $G(s) = C(sI - A)^{-1}B$ is the system transfer function and R, Q, and N are various weighting operators, then the corresponding algebraic Riccati equation and the equation for the optimal feedback operator K are

$$\begin{aligned} A^*X + XA + Q &= (B^*X + N)^*(E^*E)^{-1}(B^*X + N), \\ K &= -(E^*E)^{-1}(B^*X + N). \end{aligned}$$