Rational Approximations of Transfer Functions of Some Viscoelastic Rods, with Applications to Robust Control

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We study rational approximations of the transfer function $\widehat{P}$ of a uniform or nonuniform viscoelastic rod undergoing torsional vibrations that are excited and measured at the same end. The approximation is to be carried out in a way that is appropriate, with respect to stability and performance, for the construction of suboptimal rational stabilizing compensators for the rod. The function $\widehat{P}$ can be expressed as $\widehat{P}(s) = s^{-2}g(\beta^2(s))$, where g is an infinite product of fractional linear transformations and β is a (generally transcendental) function that characterizes a particular viscoelastic material. First, $g(\beta^2)$ is approximated by its partial products $g_N(\beta^2)$. For relevant values of β^2, convergence rates for g_N are analyzed in detail. Convergence suitable for our problem requires the introduction of a new irrational convergence factor, which must be approximated separately. In addition, the fractional linear factors in $\beta^2(s)$ that appear in $g_N(\beta^2(s))$ must be replaced by something rational. When the damping is weak it is possible to do this by separating the oscillatory modes from the "creep" modes and ignoring the latter; in general, this step remains incomplete. Some numerical data illustrating all the stages of the process as well as the final results for various viscoelastic constitutive relations are presented.

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