Quadratic Optimal Control of Stable Systems through Spectral Factorization

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We consider the infinite horizon quadratic cost minimization problem for a linear system with finitely may inputs and outputs. A common approach to treat a problem of this type is to construct a semigroup in an abstract state space, and to use infinite-dimensional control theory. However, this approach is less appealing in the case where there are discrete time delays in the impulse response, because such time delays force both the control operator and the observation operator to be unbounded at the same time. In order to be able to include this case we take an alternative approach. We work in an input-output framework, and reduce the problem to a symmetric Wiener-Hopf problem, that can be solved by means of a canonical factorization of the symbol. In a standard shift semigroup realization this amounts to factorizations of the Riccati operator and the feedback operator into convolution operators and projections. Our approach leads to a new significant discovery: in the case where the impulse response of the system contains discrete time delays, the standard Riccati equation is incorrect; to get the correct Riccati equation one must partially replace the feed-through matrix of the system by the feed-through matrix of the spectral factor. This means that, before it is possible to even write down the correct Riccati equation, one must first solve a spectral factorization problem to find one of the weighting matrices in this equation.