# Passive and Conservative Continuous Time Impedance and Scattering Systems. Part I: Well-Posed Systems

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Let $U$ be a Hilbert space. By a $\mathcal{L}(U)$*-valued positive analytic function* on the open right half-plane we mean an analytic function which satisfies the condition $\widehat{\mathfrak{D}} + \widehat{\mathfrak{D}}^* \geq 0$. This function need not be *proper*, i.e., it need not be bounded on any right half-plane. We study the question under what conditions such a function can be realized as the transfer function of an *impedance passive system*. By this we mean a continuous time state space system whose control and observation operators are not more unbounded than the (main) semigroup generator of the system, and in addition, there is a certain energy inequality relating the absorbed energy and the internal energy. The system is (impedance) *energy preserving* if this energy inequality is an equality, and it is *conservative* if both the system and its dual are energy preserving. A typical example of an impedance conservative system is a system of hyperbolic type with collocated sensors and actuators. We give several equivalent sets of conditions which characterize when a system is impedance passive, energy preserving, or conservative. We prove that a impedance passive system is well-posed if and only if it is proper. We furthermore show that the so called *diagonal transform* (which is a particular rescaled feedback/feedforward transform) maps a proper *impedance* passive (or energy preserving or conservative) system into a (well-posed) *scattering* passive (or energy preserving or conservative) system. This implies that, just as in the finite-dimensional case, if we apply negative output feedback to a proper impedance passive system, then the resulting system is (energy) stable. Finally, we show that every proper positive analytic function on the right half-plane a (essentially unique) well-posed impedance conservative realization, and it also has a minimal impedance passive realization.

**Keywords:** Dissipative, energy preserving, proper, collocated sensors and actuators, positive real, Caratheodory-Nevanlinna function, Titchmarsh-Weyl function, bounded real lemma, Kalman-Yakubovich-Popov lemma, diagonal transform.