

# *J*-Energy Preserving Well-Posed Linear Systems

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This is a short survey of the notion of a well-posed linear system. We start by describing the most basic concepts, proceed to discuss dissipative and conservative systems, and finally introduce *J*-energy preserving systems, i.e., systems that preserve energy with respect to some generalized inner products (possibly semi-definite or indefinite) in the input, state, and output spaces. The class of well-posed linear systems contains most linear time-independent distributed parameter systems: internal or boundary control of PDE:s, integral equations, delay equations, etc. These systems have existed in an implicit form in the mathematics literature for a long time, and they are closely connected to the scattering theory by Lax and Phillips and to the model theory by Sz.-Nagy and Foiaş. The theory has been developed independently by many different schools, and only recently have these different approaches begun to converge. One of the most interesting objects of present study is the Riccati equation theory for this class of infinite-dimensional systems ( $H^2$ - and  $H^\infty$ -theory).