Feedback Representations of Critical Controls for Well-Posed Linear Systems

Olof J. Staffans
Åbo Akademi University
Department of Mathematics
FIN-20500 Åbo, Finland
Olof.Staffans@abo.fi
http://www.abo.fi/~staffans/

Keywords: Suboptimal H^∞ control, (J,S)-spectral factorization, (J,S)-inner-outer factorization, (J,S)-inner coprime factorization, (J,S)-lossless coprime factorization.

This is the first part in a three part study on the suboptimal full information H^∞ problem for a well-posed linear system with input space U, state space H, and output space Y. We define a cost function $Q(x_0,u) = \int_{\mathbf{R}^+} \langle y(s), Jy(s)\rangle_Y \, ds$, where $y \in L^2_{\mathrm{loc}}(\mathbf{R}^+;Y)$ is the output of the system with initial state $x_0 \in H$ and control $u \in L^2_{\mathrm{loc}}(\mathbf{R}^+;U)$, and J is a self-adjoint operator on Y. The cost function Q is quadratic in x_0 and u, and we suppose (in the stable case) that the second derivative of $Q(x_0,u)$ with respect to u is nonsingular. This implies that, for each $x_0 \in H$, there is unique critical control u^{crit} such that the derivative of $Q(x_0,u)$ with respect to u vanishes at $u = u^{\mathrm{crit}}$. We show that u^{crit} can be written in feedback form whenever the input/output map of the system has a coprime factorization with a (J,S)-inner numerator; here S is a particular self-adjoint operator on U. A number of properties of this feedback representation are established, such as the equivalence of the (J,S)-losslessness of the factorization and the positivity of the Riccati operator on the reachable subspace.