Stabilization by Collocated Feedback

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Recently Guo and Luo (and independently Weiss and Tucsnak) were able to prove that the damped second order system

$$\begin{aligned} \ddot{z}(t) + A_0 z(t) &= -\frac{1}{2} C_0^* C_0 \dot{z}(t) + C_0^* u(t), \\ y(t) &= -C_0 \dot{z}(t) + u(t), \end{aligned}$$

can be interpreted as a continuous time (well-posed and stable) scattering conservative system with input u, state $\begin{bmatrix} \sqrt{A_0} z \\ \dot{z} \end{bmatrix}$, and output y. Here A_0 is a positive (unbounded) self-adjoint operator on a Hilbert space Z with a bounded inverse, and C_0 is a bounded linear operator from $D(\sqrt{A_0})$ to another Hilbert space U. We show that this is a special case of the following more general result: if we apply the so called diagonal transform (which is a particular rescaled feedback/feedforward transform) to an arbitrary continuous time impedance conservative system, then we always get a scattering conservative system. In the particular case mentioned above the corresponding impedance conservative system is the undamped system

$$\begin{aligned} \ddot{z}(t) + A_0 z(t) &= \frac{1}{\sqrt{2}} C_0^* u(t), \\ y(t) &= \frac{1}{\sqrt{2}} C_0 \dot{z}(t), \end{aligned}$$

which may be interpreted as a second order system with collocated actuators and sensors.

Keywords: Scattering, impedance, conservative, passive, compatible, diagonal transform, feed-back, flow-inversion.