Conservative state-space realizations of dissipative system behaviors

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It is well known that a Schur-class function S (contractive operator-valued function on the unit disk) can be realized as the transfer function $S(z) = D + zC(I - zA)^{-1}B$ of a conservative discrete-time linear system ($x(n+1) = Ax(n) + Bu(n), y(n) = Cx(n) + Du(n)$ with $U = \left[\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right]$ unitary). One method of proof of this result (the "lurking isometry" method) identifies a solution U of the problem as a unitary extension of a partially defined isometry V determined by the problem data. Reformulated in terms of the graphs of V and U, solutions are identified with embeddings of an isotropic subspace of a certain Kreĭn space $\mathcal{K}$ constructed from the problem data into a Lagrangian subspace (maximal isotropic subspace of $\mathcal{K}$). The contribution here is the observation that this reformulation applies to other types of realization problems as well, e.g., realization of positive-real or J-contractive operator-valued functions over the unit disk (respectively over the right half plane) as the transfer function of a discrete-time (respectively, continuous-time) conservative system, i.e., an input-state-output system for which there is a quadratic storage function on the state space for which all system trajectories satisfy an energy-balance equation with respect to the appropriate supply rate on input-output pairs. The approach allows for unbounded state dynamics, unbounded input/output operators and descriptor-type state-space representations where needed in a systematic way. These results complement recent results of Arov-Nudelman, Hassi-de Snoo-Tsekanovskiĭ, Belyi-Tsekanovskiĭ and Staffans and fit into the behavioral frameworks of Trentelman-Willems and Georgiou-Smith.

Keywords: Energy balance, scattering-conservative, impedance-conservative, discrete and continuous time, distributed-parameter system.