Transfer functions of regular linear systems PART II: THE SYSTEM OPERATOR AND THE LAX–PHILLIPS SEMIGROUP

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This paper is a sequel to a paper by the second author on regular linear systems (1994) referred to here as "Part I". We introduce the system operator of a well-posed linear system, which for a finite-dimensional system described by $\dot{x} = Ax + Bu$, $y = Cx + Du$ would be the s-dependent matrix $S_\Sigma(s) = \left[\begin{smallmatrix} A-sI & B \\ C & D \end{smallmatrix}\right]$. In the general case, $S_\Sigma(s)$ is an unbounded operator and we show that it can be split into four blocks, as in the finite-dimensional case, but the splitting is non-unique (the upper row consists of the uniquely determined blocks $A - sI$ and B, as in the finite-dimensional case, but the lower row is more problematic). For weakly regular systems (which are introduced and studied here), there exists a special splitting of $S_\Sigma(s)$ where the right lower block is the feedthrough operator of the system. Using $S_\Sigma(0)$, we give representation theorems which generalize those from Part I to well-posed linear systems and also to the situation when the "initial time" is $-\infty$. We also introduce the Lax-Phillips semigroup $\mathfrak{T}$ induced by a well-posed linear system, which is in fact an alternative representation of a system, used in scattering theory. Our concept of a Lax-Phillips semigroup differs in several respects from the classical one, for example, by allowing an index $\omega \in \mathbb{R}$ which determines an exponential weight in the input and output spaces. This index allows us to characterize the spectrum of A and also the points where $S_\Sigma(s)$ is not invertible, in terms of the spectrum of the generator of $\mathfrak{T}$ (for various values of ω). The system Σ is dissipative if and only if $\mathfrak{T}$ (with index zero) is a contraction semigroup.

Keywords: Well-posed linear system, (weakly) regular linear system, operator semigroup, system operator, generating operators, well-posed transfer function, scattering theory, Lax-Phillips semigroup.