Quadratic Optimal Control of Well-Posed Linear Systems

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We study the infinite horizon quadratic cost minimization problem for a well-posed linear system in the sense of Salamon and Weiss. The quadratic cost function that we seek to minimize need not be positive, but it is convex and bounded from below. We assume the system to be jointly stabilizable and detectable, and give a feedback solution to the cost minimization problem. Moreover, we tie this solution to the computation of either a (J,S)-inner or an S-normalized coprime factorization of the transfer function, depending on how the problem is formulated. We apply the general theory to get factorization versions of the bounded and positive real lemmas. In the case where the system is regular it is possible to show that the feedback operator can be expressed in terms of the Riccati operator, and that the Riccati operator is a stabilizing self-adjoint solution of an algebraic Riccati equation. This Riccati equation is nonstandard in the sense that the weighting operator in the quadratic term differs from the expected one, and the computation of the correct weighting operator is a nontrivial task.