Transfer functions of regular linear systems
PART III: INVERSIONS AND DUALITY

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We study four transformations which lead from one well-posed linear system to another: time-inversion, flow-inversion, time-flow-inversion and duality. Time-inversion means reversing the direction of time, flow-inversion means interchanging inputs with outputs, while time-flow-inversion means doing both of the inversions mentioned before. A well-posed linear system Σ is time-invertible if and only if its operator semigroup extends to a group. The system Σ is flow-invertible if and only if its input-output map has a bounded inverse on some (hence, on every) finite time interval $[0,\tau]$ ($\tau>0$). This is true if and only if the transfer function of Σ has a uniformly bounded inverse on some right half-plane. The system Σ is time-flow-invertible if and only if on some (hence, on every) finite time interval $[0,\tau]$, the combined operator Σ_τ from the initial state and the input function to the final state and the output function is invertible. This is the case, for example, if the system is conservative, since then Σ_τ is unitary. Time-flow-inversion can sometimes, but not always, be reduced to a combination of time- and flow-inversion. We derive a surprising necessary and sufficient condition for Σ to be time-flow-invertible: its system operator must have a uniformly bounded inverse on some left half-plane. Finally, the duality transformation is always possible. We show by some examples that none of these transformations preserves regularity in general. However, the duality transformation does preserve weak regularity. For all the transformed systems mentioned above, we give formulas for their system operators, transfer functions and, in the regular case and under additional assumptions, for their generating operators.

Keywords: Well-posed linear system, regular linear system, operator semigroup, system operator, time-inversion, flow-inversion, time-flow-inversion, dual system, conservative system, Lax–Phillips semigroup.