A Riccati equation approach to the standard infinite-dimensional H^∞ problem

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We solve the standard H^∞ problem (also called the H^∞ four-block problem, or the H^∞ measurement feedback problem, or the general regulator problem) for regular well-posed linear systems (in the sense of George Weiss). We give the solution in terms of two algebraic Riccati equations and the corresponding coupling condition, or alternatively, in terms of two nested algebraic Riccati equations.

The dynamics of our system is described by the system of equations

$$\begin{cases} x'(t) = Ax(t) + B_1u(t) + B_2w(t) & \text{in } D(A^*)^*, \\ z(t) = (C_1)_{\mathrm{w}}x(t) + D_{11}u(t) + D_{12}w(t) & \text{in } Y, \\ y(t) = (C_2)_{\mathrm{w}}x(t) + D_{21}u(t) + D_{22}w(t) & \text{in } Z, \end{cases} \tag{1}$$

for almost all $t > 0$, with initial state $x_0 \in H$, disturbance input $w \in L^2_{\mathrm{loc}}(\mathbf{R}_+;W)$, control input $u \in L^2_{\mathrm{loc}}(\mathbf{R}_+;U)$, regulated output $z \in L^2_{\mathrm{loc}}(\mathbf{R}_+;Z)$ and measurement output $y \in L^2_{\mathrm{loc}}(\mathbf{R}_+;Y)$ (which is used as an input to the controller). Here $\left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right]$ are the generating operators of a regular well-posed linear system, and $(C_k)_{\mathrm{w}}x := \mathrm{w}-\lim_{\alpha\to+\infty} C_k\alpha(\alpha-A)^{-1}x$ is the weak Yoshida–Weiss extension of the output operator C_k ($k = 1,2$). The state space H and the input and output spaces $\left[\begin{smallmatrix} U \\ W \end{smallmatrix}\right]$ and $\left[\begin{smallmatrix} Z \\ Y \end{smallmatrix}\right]$ may be Hilbert spaces of arbitrary dimensions.

Our main result is an infinite-dimensional version of the following standard result: there exist a dynamic controller with input y and output u which makes the closed loop system exponentially stable and also makes the norm of the mapping from w to z less than a predefined constant $\gamma > 0$ if and only if two algebraic Riccati equations have exponentially stabilizing solutions P_X and P_Y, respectively, and the spectral radius of P_XP_Y is less that γ^2. Another equivalent condition in terms of two nested Riccati equations is available as well. Finally, we establish a generalized version of the standard parameterization of all stabilizing solutions. The exact formulation varies depending on the regularity assumptions that we make, but our assumptions allow for roughly twice as much unboundedness of the control and observation operators as the Pritchard–Salamon class does, and they permit a countable number of pure delays in the input/output responses.