Passive and Conservative Infinite-Dimensional Linear State/Signal Systems

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We develop the theory of linear infinite-dimensional passive and conservative time-invariant systems in discrete and continuous time. The model that we use is built around a state/signal node, which differs from a standard input/state/output node in the sense that we do not distinguish between input singnals and output signals, only between the "interior" state space and the "exterior" signal space. Our state/signal model is an infinite-dimensional version of Willem's behavioral model with latent variables interpreted as the state.

In this short abstract we limit ourselves to the discrete time case.

Definition 0.1. By a *state/signal node* we mean a triple $\Sigma = (V, \mathcal{X}, \mathcal{W})$, where the *state space* $\mathcal{X}$ and the *signal space* (exterior space) $\mathcal{W}$ are Hilbert spaces, and V is a subspace of the product space $\mathfrak{K} := \begin{bmatrix} \mathcal{X} \\ \mathcal{X} \\ \mathcal{W} \end{bmatrix}$ with the following properties:[1]

[1]Later when we introduce *passive* nodes we shall require $\mathcal{X}$ to be a Hilbert space, $\mathcal{W}$ to be a Kreĭn space, and equip $\mathfrak{K}$ with a particular Kreĭn space structure rather than the Hilbert space structure that it inherits from $\mathcal{X}$ and $\mathcal{W}$.

(i) V is closed in $\mathfrak{K}$;

(ii) For every $x \in \mathcal{X}$ there is some $\left[\begin{smallmatrix} z \\ w \end{smallmatrix}\right] \in \left[\begin{smallmatrix} X \\ \mathcal{W} \end{smallmatrix}\right]$ such that $\left[\begin{smallmatrix} z \\ x \\ w \end{smallmatrix}\right] \in V$;

(iii) If $\left[\begin{smallmatrix} z \\ x \\ w \end{smallmatrix}\right] \in V$ and $\left[\begin{smallmatrix} x \\ w \end{smallmatrix}\right] = \left[\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}\right]$, then $z = 0$;

(iv) The set $\left\{ \left[\begin{smallmatrix} x \\ w \end{smallmatrix}\right] \in \left[\begin{smallmatrix} \mathcal{X} \\ \mathcal{W} \end{smallmatrix}\right] \;\middle|\; \left[\begin{smallmatrix} z \\ x \\ w \end{smallmatrix}\right] \in V \text{ for some } z \in \mathcal{X} \right\}$ is closed in $\left[\begin{smallmatrix} \mathcal{X} \\ \mathcal{W} \end{smallmatrix}\right]$.

All of these conditions have a clear meaning, related to the role played by V in our definition of a trajectory. By a *trajectory* $(x(\cdot), w(\cdot))$ *along* V on $\mathbb{Z}^+ = 0, 1, 2, \ldots$ we mean a pair of sequences $\{x(n)\}_{n\in\mathbb{Z}^+}$ and $\{w(n)\}_{n\in\mathbb{Z}^+}$ satisfying

$$\begin{bmatrix} x(n+1) \\ x(n) \\ w(n) \end{bmatrix} \in V, \; n \in \mathbb{Z}^+. \tag{1}$$

We explain what standard notions, such as existence and uniqueness of solutions, continuous dependence on initial data, observability, controllability, stabilizability, detectability, and stability mean in this setting. Out of these especially our notion of (approximate) controllability seems to be new in a behavioral context. We show that each state/signal system has three types of representations (none of which is unique): a *latent variable* representation, a *kernel representation*, and a *input/state/output* representation. We also define the notion of *transfer function* for each of the three types of representations.

We next replace the signal space $\mathcal{W}$ by a Kreĭn space, and look more closely at systems that are simple and conservative or minimal and passive. In particular, at this stage we also define the dual of a state/signal system. We equip the space $\mathfrak{K}$ with the Kreĭn space inner product

$$\left[\left[\begin{smallmatrix} z_1 \\ x_1 \\ w_1 \end{smallmatrix}\right], \left[\begin{smallmatrix} z_2 \\ x_2 \\ w_2 \end{smallmatrix}\right]\right]_{\mathfrak{K}} = -\langle z_1, z_2\rangle_{\mathcal{X}} + \langle x_1, x_2\rangle_{\mathcal{X}} + [w_1, w_2]_{\mathcal{W}}, \tag{2}$$

For the dual state/signal system we use a sligtly different Kreĭn space inner product, namely

$$\left[\left[\begin{smallmatrix} z_1 \\ x_1 \\ w_1 \end{smallmatrix}\right], \left[\begin{smallmatrix} z_2 \\ x_2 \\ w_2 \end{smallmatrix}\right]\right]_{\mathfrak{K}_*} = -\langle z_1, z_2\rangle_{\mathcal{X}} + \langle x_1, x_2\rangle_{\mathcal{X}} - [w_1, w_2]_{\mathcal{W}}. \tag{3}$$

We identify the dual of $\mathfrak{K}$ with $\mathfrak{K}_*$ by using the duality pairing

$$\left[\left[\begin{smallmatrix} z \\ x \\ w \end{smallmatrix}\right], \left[\begin{smallmatrix} z_* \\ x_* \\ w_* \end{smallmatrix}\right]\right]_{[\mathfrak{K},\mathfrak{K}_*]} = -\langle z, x_*\rangle_{\mathcal{X}} + \langle x, z_*\rangle_{\mathcal{X}} - [w, w_*]_{\mathcal{W}}, \tag{4}$$

and we always compute orthoghonal complements with respect to this duality pairing. Thus, in particular, the orthogonal complement $V^{[\perp]}$ of $V \subset \mathfrak{K}$ is a subspace of $\mathfrak{K}_*$. The dual state/signal system is $\Sigma_* = (V_*, \mathcal{X}, \mathcal{W}_*)$, where $V_* = V^{[\perp]}$ and $\mathcal{W}_* = -\mathcal{W}$. A system $\Sigma = (V, \mathcal{X}, \mathcal{W})$ is *passive* if V is a maximal positive subspace of $\mathfrak{K}$, it is *energy preserving* if V is an isotropic, maximal positive subspace of $\mathfrak{K}$, and it is *conservative* if V is a Lagrangean subspace of $\mathfrak{K}$. All of these notions can be characterized in a number of other equivalent ways. As usual, Σ is passive or conservative if and only if the dual system $\Sigma_* = (V_*, \mathcal{X}, \mathcal{W}_*)$ is passive or conservative, respectively.

By looking at a conservative or passive state/signal system from different points of view (i.e., by splitting the signal space $\mathcal{W}$ into the direct sum $\mathcal{W} = \mathcal{U} \dot{+} \mathcal{Y}$ of an input space $\mathcal{U}$ and an output space $\mathcal{Y}$ in different ways) we recover the well-know scattering, impedance, and transmission input/state/output settings. The family of different scattering (or impedance or transmission) systems that we obtain in this way is the orbit of one fixed scattering system under linear fractional transformations whose coefficient matrices are (J_1, J_2)-unitary, where the choice of J_1 depends on the setting (equal to $\left[\begin{smallmatrix} I & 0 \\ 0 & -I \end{smallmatrix}\right]$ in the scattering case) and $J_2 = \left[\begin{smallmatrix} I & 0 \\ 0 & -I \end{smallmatrix}\right]$ is the signature operator corresponding to the fixed scattering system. A similar statement is also true for the transfer functions of these systems (which in the scattering case are known under the name *scattering matrices*).

We construct concervative, minimal optimal, minimal $*$-optimal, and balanced passive realization of a given passive transfer function. All of these realizations are unique up to unitary similarity. The balanced realization is obtained by interpolating half-way between the minimal optimal and the minimal $*$-optimal realization.

We pay special attention to the case where the transfer function is lossless from one or two sides, connecting this property to the strong one-sided or two-sided stability of the main semigroup of the simple conservative scattering representation, and to the one-sided or two-sided strong conditional stability of the simple conservative state/signal realization.

The presentation by D. Arov entitled “Reciprocal passive linear time-invariant systems” is based on the technique developed here.